SECURING THE ENVIRONMENT AND SECURING STATES

Urs Luterbacher and Carla Norrlöf

INTRODUCTION

Is conflict driven by environmental scarcities or an abundance of natural resources? For quite some time, this question has generated a lively academic debate. The theoretical literature and empirical evidence it offers are inconclusive. On the one hand, authors such as Homer-Dixon (1994) have emphasized the importance of resource scarcities in explaining conflict. On the other hand, scholars such as Collier and Hoeffler (1998) have tried to link conflict with a relative abundance of natural resources. We believe that the failure to provide a coherent explanation upon which rigorous predictions can be based is due to the neglect of institutions in understanding resource use. What we will try to highlight here is the importance of institutional settings to explain this apparent paradox.

Ever since Hardin's seminal article on "The Tragedy of the Commons," the importance of property rights for preventing the overuse of natural assets has been widely acknowledged (Hardin, 1968). Whereas open access to resources leads to overexploitation and abuse, proper regulation generates efficiency and conservation. As a major corollary, the question of environmental scarcities cannot be reduced to exogenous "natural forces." If overdrawn, an overabundance of unregulated resources can very quickly give way to scarcity. Similarly, a country blessed with a copious

War, Peace and Security
Contributions to Conflict Management, Peace Economics and Development, Volume 6, 267–296

ISSN: 1572-8323/doi:10.1016/S1572-8323(08)06016-5

amount of precious resources may experience shortages after a military ruler or warlord seizes them to appropriate other scarce and unprotected resources. In other words, if overuse triggered the scarcity that provoked conflict, institutional deficiencies are at the root of conflict, even if such conflict emanates from the dearth of natural resources. Poor institutions may also contribute to conflict when attempts are made to capture a precious good (such as minerals or timber) that exists in plenty in a particular area. Using force to control that particular resource can then facilitate violent appropriation of other scarce assets within a society. In either case, a society incapable of instituting a proper regulatory framework will see the eruption of resource conflict.

Environment-related conflicts are but one way to think about connections between the environment and organization and power structures within and between societies. A regulatory scheme is necessary to ensure both environmentally sustainable and well-organized, well-run societies. The power to enforce property rights often requires coercion both to deter predatory moves within, just as importantly between, states. A well-managed society requires regular means to maintain a coercive apparatus. Enforceable property rights are essential to economic activity, and it is necessary to maintain a production base in order to raise the taxes to fund a coercive apparatus capable of enforcing such rights (North, 1981). This connection between spheres of production and protection also has important implications for international relations. Efficient resource use, which ultimately is tied to property rights definitions, determines international structures much more generally.

We develop a stylized model of society that provides a private good for production purposes and collective good for defense purposes. We examine its characteristics first as a single (autarchic) society and then as an entity of an international system. We will show in particular how the taxation base of a society, as well as some other basic parameters related to defense and military technology, is paramount in determining its military posture toward other societies. Whereas technological factors are exogenously determined, two phenomena largely influence the taxation base and domestic policies. First is the extent to which a society is able to organize itself efficiently, i.e., in more technical terms to reach Pareto optimality. This is in our view another metaphor for well-defined regulatory schemes and property rights structures. The second important factor enhancing the taxation base is the existence of a significant amount of transaction both within and among societies. In fact, these two aspects reinforce each other: an efficient (Pareto optimal) organization encourages transactions within a society, which then can lead to additional transactions between societies in the form of trade.

Our model is based on a modified version of the theory on the optimal use of exhaustible and slowly renewable resources advanced by Dasgupta and Heal (1979). We have made adjustments to take into account variations in returns to scale in production and protection, as well as in the transformation rate between these activities, and for the possibility that actors may be risk preferring in their consumption and production attitudes and decision making more generally (Fishburn & Kochenberger, 1979; Kahneman & Tversky, 1979; Dacey, 1998). These changes are necessary to generate the kind of relationships outlined above. They reflect our criticism of alternative conceptions of the relationship between production and protection, in particular, the Konrad–Skaperdas conception, which is based on a burgeoning literature on the origin of power structures and the market to protect property rights (cf. Grossman, 1991; Hirshleifer, 1988, 1989, 1995; Usher, 1989). The Konrad–Skaperdas approach is an examination of simple agricultural-type productive structures that are threatened by raids of "bandits." According to their analysis, the need for protection can generate one of four power structures: *Anarchy*, an equilibrium in which peasants rely on individual protection against bandits; *Self-governance* in which peasants organize a common protection force against bandits; *Leviathan* in which an individual ruler takes a "cut" from peasant revenue in exchange for protection; and *Competing Lords* who each take a cut from peasant revenue in exchange for protection.[1] Since these will end up fighting each other in the competition for protection, the equilibrium situation they generate is sub-optimal and referred to as the "tragedy of coercion." Which power structure eventually prevails, in this model, is determined by changes in the relative costs between private and collective protection. Anarchy results if the difference between private and collective protection is low. Otherwise some form of state structure – Leviathan, Self-governance, or a society of Competing Lords – is bound to emerge. The cost of protection – specifically changes in the unit cost of soldiers – faced by the "Leviathan" ruler or "Competing Lords" is also decisive. Whenever wages for soldiers are lower than peasants, a Leviathan regime will materialize at first, then one of Competing Lords, but never one of Self-governance.

The Konrad–Skaperdas conception is not only based on the assumption of diminishing returns to agricultural production, but also on the general notion that major trade-offs exist between productive and protective activities and between private and collective defense. All these assumptions are debatable. First, even though agricultural production is subject to diminishing returns in the long-term, intensifying production or opening new lands through the adoption of irrigation techniques or more aggressive cultivation methods can increase relative yields. Second, coercive methods almost always have been used to open up new land for cultivation and herding, underlining the strong complementarities between protection and agricultural production. Over time, it will also become rational to invest in training and weaponry. At some point, a militia capable of protecting productive activities from predatory bandits and power hungry lords will appear. Finally at some level of production, trade in agricultural products and later in manufactured goods will begin.

Collective Action and Dual Strategy

The main drawback of the Konrad–Skaperdas model of society is that it is mainly driven by the relative costs of private versus collective protection and thus mostly by the evolution of military technology. Other scholars adopting similar perspectives, such as Hirshleifer's, adopt similar assumptions.[2] There is either very little qualitative or quantitative evidence suggesting that the possession of more advanced military technology ultimately carries the day except in situations of extreme asymmetries such as the conquest of Central and South America by Spain or the colonial wars of the nineteenth century. Rockets and bombs did not prevent the Mongolian conquest of China, and the Byzantine Empire continued to decline even after the invention of Grecian fire. We suggest, rather, that the interactions between production and protection are essential in determining the characteristics of states and state systems. Moreover, these are not recent developments but date far back in human history. Archeologists tell us that the first states were established to solve certain types of resource problems related to the adoption of large-scale agricultural technologies and the promotion and protection of trade routes.[3] Increasing returns to scale could be achieved by investments in massive collective works like irrigation networks, which also necessitated the building of canals, dams, and reservoirs. This required an organization of production and labor that went beyond the local level. Moreover, if a surplus was achieved, storage facilities were needed. These new activities required coordination and supervision. At the same time, the surplus and infrastructures attracted outsiders. Management and protection of agricultural production led ultimately to the creation of specialized armed units. In time, these forces might try to gain a larger share of the surplus by repressing locals, by conquering new land, or by protecting trade routes and distant colonies. Since benefits can be made from raiding other states' resource reserves, specialized raiding states with appropriate armed forces will emerge. One can also imagine the creation of states that specialize in the protection of others and reap benefit from this, for example Sparta or Rome. As history has shown, such systems easily evolve into hegemonic ones.

Typically, the organization of one collective good – new agricultural technologies or irrigation – therefore results in other collective goods – defense, armed forces, and organized trade. This stylized presentation clarifies the connection among different collective goods linked to state-building and makes it abundantly clear that states have dual strategic goals. On the one hand, they must constantly tend to their resource base, and on the other hand, they must protect that resource base in order to ensure its long-term viability. To present a state uniquely as a security seeker is thus a gross misrepresentation of its fundamental aspects.[4]

Although a state is usually characterized as a particular kind of collective good, previous work has focused largely on the difficulties of "purchasing" the collective good, group size, and free rider effects (Olson, 1965, 1982; Sandler, 1992). The link between property rights and collective goods, so-called commons, has also been analyzed widely (Hardin, 1968; Ostrom, 1990). But the pervasive links between protection and production that exist within society and the way these "dual strategies" shape patterns of interaction at the international level largely have been ignored. A notable exception here is Gilpin's War and Change in World Politics (Gilpin, 1981). In order to show how change occurs in the international system, Gilpin uses an S-curve to model the expansion of a particular country and argues for the importance of property rights protection in determining the international distribution of wealth, but he does not provide a framework that connects property rights protection with military protection within a society and therefore cannot tell us how the relationship between property rights protection and military protection shapes interactions between states. In this study, we are interested precisely in the domestic and systemic effects of balancing private and public goods. The main defining feature of a collective good is "jointness of supply" – once supplied to one actor, the good is supplied to all. The ability of a particular actor to use the collective good cannot be attributed exclusively and will depend on group size (how many others are attempting to do the same). A perfect jointness implies how much one actor consumes will not affect others' level of consumption; although, jointness is often so imperfect that some rivalry is involved. By contrast, a private good is not characterized by jointness and can be excluded. Therefore, an individual's utility level is uniquely affected by its own consumption. But whenever the good in question is collective, jointness is often imperfect and exclusion prohibitively costly. How much others consume therefore affects the individual utility levels. In their classical work *Economic Theory and Exhaustible Resources*, Dasgupta and Heal show that "jointness" is derived from a general theory of externalities, which can either be positive or negative. The theory of externalities they presented is based on the simplifying assumption of the availability or production of two goods, one private and one collective, within a socio-economic system. This could also represent a political structure composed of a productive (private good) sector assuring subsistence and a government with a defense organization. As we will show, this theory of externalities is remarkable in its ability to give meaning to a wide range of social phenomena as part of one theoretical construction. The establishment of a formal theory of externalities should help clarify collective good and property rights issues and correct some of the weaknesses in the literature on "conflict and exchange."

THE MODEL

The formal theory of externalities assumes that for N individuals (or households) in a particular social group g, x_i represents the quantity of the private good consumed by individual i and $g_1, g_2, g_3, \ldots, g_i, \ldots, g_N$, the amounts of the collective good used by individuals $1, \ldots, i, \ldots, N$ (Dasgupta & Heal, 1979).

$$u_i = u_i(x_i, g_1, \ldots, g_i, \ldots, g_N) \tag{1}$$

$$u_i = u_i\left(x_i, \sum_{j=1}^{N} g_i\right) \tag{2}$$

The expression (2) is a special case of (1) in which individual (or household) i's utility depends on the total quantity of the collective good consumed, purchased, or produced by everyone (Dasgupta & Heal, 1979). Let us further assume that either all individuals are identical or very similar in their preferences or agent i represents a median decision maker that sets the tone for what is happening in society. A crucial assumption resides now in the definition and specification of u_i. Standard assumptions about rational behavior contradict empirical evidence by assuming that most preference schemes, whether individual or collective, can be described as either risk neutral or averse. These premises are established usually for mathematical convenience to simplify complex issues and reduce them to simple linear ones. Experimental psychologists and even observers of animal behavior have noticed that risk preference often follows risk aversion in situations where decision makers are faced with the prospect of loss (Stephens, 1990). Risk aversion and preference are usually seen together, and various attempts have been made to explain their joint appearance. The principal analyzes of hybrid risk attitudes are given by Battalio, Kagel, and Jiranyakul (1990), Battalio, Kagel, and MacDonald (1985), Camerer (1989), Fishburn and Kochenberger (1979), and Kahneman and Tversky (1979).[5] In particular, Fishburn and Kochenberger (1979) show that the majority of individuals have everywhere an increasing utility function $u(x)$, where x is a measure of gains and losses that increases more than proportionally for small or negative x and eventually increases less than proportionally for relatively high values of x. *Most individuals are thus risk-averse over gains and risk preferring over losses*. This notion can serve as a theoretical justification for the contention elaborated by Hirshleifer (1991a) that the poor have a comparative advantage in appropriation, obviously a more risky way to acquire wealth than capital accumulation through savings.

A natural extension of these considerations is to represent an average decision maker's utility function by an everywhere increasing S-curve in x, which adequately expresses the mix of risk aversion under gains and preference over losses.[6] An S-curved utility function does not just obtain as a result of psychological analysis. It may also result from the following: we can imagine a locally non-satiated representative agent as the only producer and consumer in a competitive economy, and all relative prices are supposed positive, the aggregate demand for every variety of the commodity must equal its aggregate supply. Since we postulate only one representative agent, we get: $c_i^{\mathrm{D}} = c_i^{\mathrm{S}}$, where c_i^{S} is the produced (and supplied) amount of commodity i. Thus, the representative agent has the following utility function: $u = \int_{i=1}^{n} c_i^{\mathrm{D}} dc$, where c_i^{D} is the demanded amount of a variety of the only consumption good.

We can further postulate that the utility function is strictly monotonic in all varieties of consumption good and the agent basically consumes what he produces. We can thus, in our analysis, focus exclusively on the production function of the goods. In order to maximize his utility, our agent simply maximizes production. This production function can exhibit at first increasing then decreasing returns with respect to its arguments.

Suppose that there is only one variable factor of production and one commodity is produced. This production function expresses the plausible assumption that initial increases in the level of productive activity will generate more than proportional returns in the production good c_i^S, but then eventually, with further increases in inputs, less than proportional output will appear. Clearly, in this case as well, the agent's utility function can be expressed by an S-shaped curve located somewhat differently in the utility-valued item plane than the one resulting from the psychological approach: The minimum level of the S-curve is then clearly located at the origin. If we then conceive of several agents, each of them living originally in autarchy, we can again imagine conflict and bargaining processes between these as they either cooperate with each other (trade) or fight over resources. Without loss of generality, we can then present the following risk averse/risk preferring (S-shaped) utility curve, closer to the production–protection logic.

As can be seen in Fig. 1, the function $\exp(\alpha - 1/f(x))$ has the S-curve characteristics explained above. Consistent with this hypothesis, u_i can be rewritten along the lines of expression (3) below:

$$u_i\left(x_i, \sum_{j=1}^{N} g_i\right) = \exp\left(\alpha - \frac{1}{x_i} - \frac{1}{\sum_{j=1}^{N} g_i}\right) \tag{3}$$

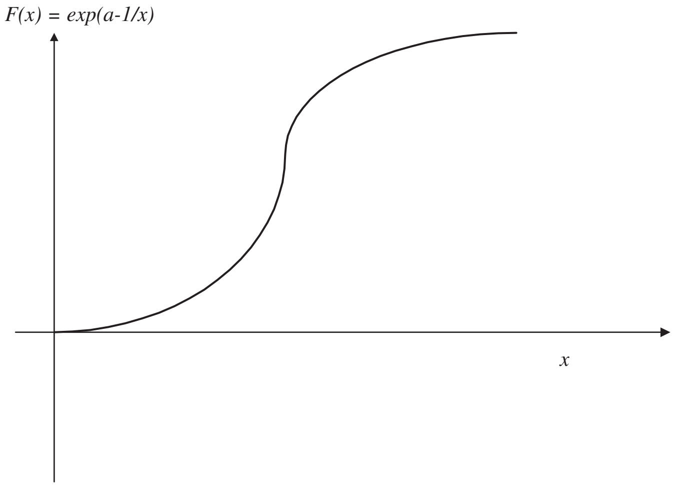

Fig. 1. The Supply of Private and Collective Goods with Hybrid Risk Preferences.

Both private and collective goods are essential for the utility of agent i. If the value of one of the goods goes to zero, the value of the whole utility function goes to zero; private goods can never be completely substituted for collective goods or vice versa. Moreover, as we will show below, the S-curve perspective (and thus the risk averse/risk preferring representation) also has an important implication in terms of the provision of collective goods; namely, the rate of conversion of private into collective goods becomes an important determinant of its supply.

Initially, individuals have one unit of private good x_i and none of collective good g_i (Dasgupta & Heal, 1979, p. 41). Agents however are able to convert the private good into collective good at the rate p^s. If $s = 1$, the private good can be transformed into collective good with constant returns to scale; if $s < 1$, the conversion takes place with increasing returns to scale; if $s > 1$ with decreasing returns to scale. With the collective good g_i, representing national defense, s represents a measure of society's ability to mobilize resources for war. The lower s is, the greater are the prospects for a society to mobilize resources.

Anarchic Equilibrium

Agent i in society g maximizes u_i as defined in Eq. (3) subject to the following budget constraint:

$$p^s g_i + x_i \leq 1 \tag{4}$$

If all agents in society maximize utility in the same way i does, based upon some expectation of the amount of the collective good produced or purchased by every other agent, a particular kind of Nash equilibrium obtained for the society in question. Dasgupta and Heal call this a *society market* or *anarchic equilibrium*. It is characterized by a pure competitive equilibrium for private goods and a non-competitive but decentralized one for collective goods. Between societies, this leads to an *international equilibrium* in which only some societies are able to organize themselves in terms of Pareto optimal Nash equilibrium. The ability to form Pareto optimal societies can be interpreted as an ability to enforce property rights and, if need be, to mobilize against other societies. As such, it is one of the main determinants of the international distribution of power. In this sense, the choice of equilibrium strategy bears directly on the structure of the international system.[7] We will return later to the question of how to organize a society in a Pareto optimal way.

In the anarchic equilibrium, every agent is ready to produce or purchase the following bundle of private and collective goods (see appendix for (A.1) and (A.2)):

$$\hat{x} = \frac{N}{\sqrt{p^s} + N} \text{ and } \hat{g} = \frac{1}{\left(\sqrt{p^s} + N\right)\sqrt{p^s}} \tag{5}$$

When N is large and p^s relatively close or equal to one, every agent keeps most of its endowment in private goods with only a small fraction devoted to the collective good.[8] We however are able to point to circumstances in which the collective good is voluntarily provided despite a large N and an abundance of private goods.[9] We do so by establishing a relationship between the conversion rate p^s and the amount of both the private and collective good that actors are willing to purchase or produce. It is entirely possible for the good to be provided through voluntary contributions in the presence of many actors and despite decreasing returns to scale (i.e., when p^s is relatively large). Moreover, under increasing returns to scale (when the conversion rate p^s is relatively low) the purchase or production of the collective good is relatively cheap and thus allows for a relatively large g per agent even if high amounts of the private good x are produced and consumed. Societies that are capable of reaping increasing returns to scale and producing high levels of private and collective goods are better equipped to compete for power and wealth at the international level.

The anarchic equilibrium is not Pareto optimal.[10] To demonstrate this, one can compute the whole exponent of the exponential function (3) at this equilibrium in terms of expression (5) which gives:

$$\frac{\left(\sqrt{p^s} + N\right)\left(\sqrt{p^s} + 1\right)}{N}$$

To compute a Pareto optimal outcome, one has to treat g as if it were a private good. As a result, agent i maximizes the following expression as if he were alone:

$$\exp\left\{\alpha - \frac{1}{x} + \frac{-1}{Ng}\right\} \text{ subject to the same budget constraint } p^s g + x \leq 1$$

The Pareto optimal solution $(\tilde{x}, \tilde{g})$ can be readily found as:

$$\tilde{x} = \frac{\sqrt{N}}{\sqrt{p^s} + \sqrt{N}}, \tilde{g} = \frac{1}{\left(\sqrt{p^s} + \sqrt{N}\right)\sqrt{p^s}} \text{ and thus, } \tilde{g} = \frac{\tilde{x}}{\sqrt{N}\sqrt{p^s}} \tag{6}$$

Again the value or the exponent of function (3) then can be computed and is:

$$\frac{\left(\sqrt{p^s}+\sqrt{N}\right)^2}{N} < \frac{\left(\sqrt{p^s}+N\right)\left(\sqrt{p^s}+1\right)}{N} \text{ (for more details refer to the appendix)}$$

The fact that the anarchic equilibrium is not Pareto optimal reflects the "tragedy of the commons" outcome where the absence or minimal provision of the collective good (here regulation) leads to a socially undesirable outcome. In fact, the difference between the anarchic equilibrium and Pareto optimal value is:[11]

$$\beta = \sqrt{p^s}\,\frac{1+\sqrt{N}\left(\sqrt{N}-2\right)}{N} > 0 \text{ for all } N > 1 \tag{7}$$

where $\beta = \beta_{\max} - \beta_{\min}$ and obviously $\beta_{\max} = \left(\left(\sqrt{p^s}+N\right)\left(\sqrt{p^s}+1\right)\right)/N$ and $\beta_{\min} = \left(\sqrt{p^s}+\sqrt{N}\right)^2/N$.

Expression (7) tells us that the anarchic equilibrium is identical with the Pareto optimal outcome whenever $N = 1$ as one would expect since it corresponds to the case where there is just one member of society or in terms of property rights one owner, who therefore has optimal provision incentives. The expression (7) above shows that the amount β measures how far a society is from achieving Pareto optimality. It also represents the degree to which a society has managed to establish a working property rights system through regulation in order to prevent open access to scarce resources. As we show elsewhere, this gap can have a significant impact on inter-regional trade patterns.

Unless an efficient market can be established that includes all externalities, Pareto optimality cannot be achieved within this anarchic structure. The creation of such a market implies, at least initially, an organization, i.e., a collective good, to define, protect, and guarantee Pareto optimality (Luterbacher, 1994). A collective endeavor, capable of concentrating coercive powers and enforcing rights, absolutely is crucial for sustaining Pareto optimal societies.

Although some scholars have seen the absence of private property rights at the domestic (Demsetz, 1967) or international level (Conybeare, 1980) as creating Pareto inefficiencies, the privatization of coercion is bound to create inefficiencies of its own. In terms of enforcing order internally, private property rights are by no means the only possible or always the most efficient form of resource management; indeed common property is often the best way to capture the increasing returns to scale and manage the transition to decreasing returns (Bromley & Feeny, 1992; Ostrom, 1990; Stevenson, 1991). This is borne out through ethnographic and historical research (Netting, 1981; Wiegandt, 1977). For centuries, warlords and governments have treated the means of coercion as a strictly private good and used force to extract rents from society. From the time when mercenaries racketed the Carthaginians or Roman soldiers and Barbarian armies did the same in the late stages of the Western Roman Empire through to our own time, past and present provide ample examples of why privatizing as means of coercion is bound to fail. Since it is impossible to get around an initial organization for the provision of a collective good, taxation has often been used to induce or force societies to maintain collective goods with regular mandatory contributions. This is the case with tax equilibrium when societies agree or are forced to maintain collective goods with regular mandatory contributions (Dasgupta & Heal, 1979, p. 54).

State Formation

Acquiring and maintaining the power to tax, however, is far from obvious. It depends on several factors: the consent of at least some of society's members, existence of relatively important levels of transactions in some form of "numeraire" that actually can be taxed, and the capability to punish recalcitrant individuals within the group. Unless a subset of society agrees to be taxed in exchange for the collective good, political entrepreneurs have to rely on their own private source of revenue.[12] Even in this case though, the collective good is likely to be supplied at a sub-optimal level, at least initially. A significant reduction in the number of taxable transactions is much more difficult to overcome and requires, if possible at all, a complete reorganization of the social order. In one of the great transformations of Europe, the Moslem raids of the eighth century AD on the Mediterranean coastline dramatically reduced domestic and "international" trade, brought down the Frankish Merovingian dynasty, replaced it with the Carolingian dynasty, and ushered in a feudal order through further invasions and even lower levels of transactions.[13]

As long as the tax is explicitly meant to correct for the Pareto inferior outcome represented by the anarchic equilibrium – in other words as long as it is "Pigouvian" – it can compensate for the failure of the anarchic equilibrium to generate a Pareto optimal outcome.[14] In some cases, however, the tax authority will be tempted to impose a levy that is higher than what is required to attain the Pareto superior equilibrium; in others it will be too weak to extract the full Pigouvian amount.

We now assume that an authority offers a subsidy t on the purchase or production of a unit of the collective good by agent i and τ, a tax (lump-sum) on i in terms of the private good.[15] This allows us to arrive at the Pareto optimal outcome in which total expenditures or subsidies for the collective good are identical (see appendix for (A.3)–(A.5)):

$$N\tilde{g}t = N\tau = \frac{N}{\left(\sqrt{p^s} + \sqrt{N}\right)\sqrt{p^s}} \frac{(N-1)p^s}{N} = \frac{(N-1)\sqrt{p^s}}{\sqrt{p^s} + \sqrt{N}} \tag{8}$$

Under Pigouvian taxation principles, total expenditures equal total revenues. Consequently, the budget is balanced for the collective good and is Pareto optimal. Expression (8) can be used to compute the optimal size in terms of N, the coalition required to arrive at the Pareto optimal tax equilibrium (see appendix). With N as a function of p^s increasing either exponentially (if $s > 1$, Fig. 2) or logarithmically (if $s < 1$), we see the coalition required to establish the collective good (see appendix). If the transformation rate from a private to a collective good can be done more than proportionally, supply of it can occur despite small numbers. Coalitions with a few large actors that enjoy increasing returns to scale can supply collective goods to many small actors in the international system experiencing decreasing returns to scale, resulting in the well known tendency for the "small to exploit the large" (cf., Olson, 1982) (Fig. 3).

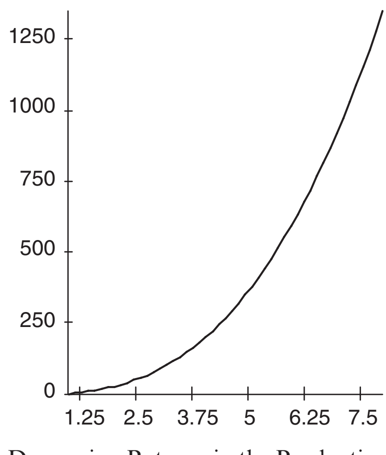

Fig. 2. Coalition Size: Decreasing Returns in the Production of the Collective Good ($s = 2.8 < 1$).

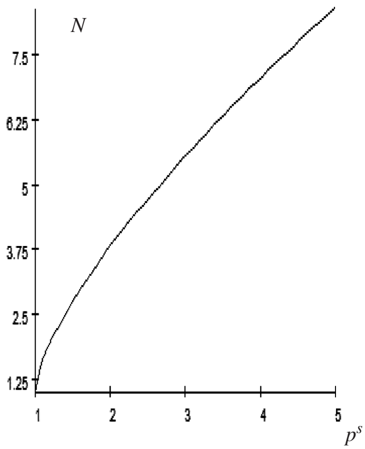

Fig. 3. Coalition Size: Increasing Returns in the Production of the Collective Good ($s = 0.6 < 1$).

State Structures and the International System

In our model, the international distribution of power is determined by the respective size of the interacting societies' defense sectors. To predominate in the international system, states must outclass competitors in two distinct ways. Internally, enforcement powers must secure property rights, and externally they must prevent a country's land and possessions from seizure by an external aggressor. If other elements impinging upon the relations between societies such as trade are ignored, one can measure the defense sectors by the equilibrium size of the collective goods established in the preceding section and their associated expenditures. To arrive at an international equilibrium, these defense sectors must be evaluated with respect to each other. Bearing in mind that Ng_i represents the defense sector of society g, then assuming a two society world, the combination of factors that a society g needs in order to overcome a society q of M members, i.e., the defense sector of Mq, will be given by a specific combination of parameters reflecting defense and offense technology. The ratio Ng over Mq therefore must exceed a certain value determined by the military technologies mastered by society g and q. Extending this logic a bit further, we can say that a given combination of defense–offense technology and training parameters constitute a constraint for the maximization of the utility function u_i for our representative agent i. This constraint can be represented as:

$$\frac{Ng_i}{Mq_i} > L_g \tag{9}$$

where L_g stands for a linear combination of minimal ratios between the forces of g and q in all the relevant aspects of military power. Dimensions that include land, naval, air power, and the ability to concentrate and disperse, which must be superseded by g's overall defense capabilities if g is to acquire dominance in combat.[16]

The utility function for agent i must be redefined in terms of the "threat" of a foreign armed force. If we assume that foreign forces have to be countered with a targeted amount of domestic forces corresponding to $\overline{\overline{Ng_i}}$, the amount $\overline{\overline{Ng_i}}$ will have a positive effect on the utility function of agent i. The utility function will then take the following form:

$$u_i\left(x_i \sum_{j=1}^{N} g_i\right) = \exp\left(\alpha - \frac{1}{x_i} - \frac{1}{\Sigma_{j\neq 1}^{N} g_j + g_i} - \frac{1}{\overline{\overline{Ng_i}}}\right) \tag{10}$$

This utility function is maximized subject to the already established budget and defense constraint given by relation (9), namely, $Ng_i/Mq_i > L_g$. This eventually (see appendix for (A.6)–(A.13) and Eq. (11)) leads to the following expression if Pareto optimality is assumed:

$$\left[\frac{L_g T}{mp^s}\right]^{1/2} \exp\frac{1}{2}\left(\alpha - \frac{1}{x_i} - \frac{1}{Ng_i} - \frac{1}{Mq_i}\right) = Ng_i \tag{11}$$

Among other things, expression (11) thus determines the optimal reaction of society g in terms of its defense sector Ng_i with respect to Mq_i, the defense sector of society q. As can be seen here, the level of Ng_i can increase under the influence of five factors:

(1) Level of taxation T.
(2) Importance of military technology represented by the L ratios, which can also be interpreted as a probability of military victory in the battlefield.
(3) Low m, the shadow price of "defense."
(4) Low p^s, which is the rate of transformation from the private into collective (defense good).
(5) Large threat as represented by Mq, the foreign defense good.

In the present terminology, the size of the defense sector of country g (Ng) is related to the size of the defense sector of country q (Mq) via an S-shaped reaction function. In expression (11), the sum of the two fractions is $-(1/x_i + 1/Ng_i)$ properly recalibrated is practically equivalent to the sum of the civilian and non-civilian sectors of an economy, thus its GDP. Moreover, expression (11) also tells us that the taxation levels, technological weapon characteristics (D_g), conversion ratio from private (civilian) to defense goods (p^s), and shadow price of defense (m), all play a role in determining armament outlays in a particular country. And since expression (11) can be thought as symmetric in the other society q, we end up with:

$$\left[\frac{L_q T_q}{m_q p_q^s}\right]^{1/2} \exp\frac{1}{2}\left(\alpha_q - \frac{1}{y_i} - \frac{1}{Mq_i} - \frac{1}{Ng_i}\right) = Ng_i \tag{12}$$

Competition for military superiority can thus be described by the two optimal reaction curves (11) and (12). Depending on marginal propensities to react, our model predicts a wide array of situations discussed in the literature on international relations and conflict and exchange.

At the international level, three scenarios are obtained. The most desirable, Pareto optimal outcome, is one where states do not react strongly toward each other. Instead, they coexist peacefully in a stable international equilibrium (see Fig. 4).

A second situation exists when states are susceptible to changes in the opponents' military capabilities, producing a stable arms race. This leads to a competition for military superiority and a situation in which all societies eventually reach a limit to the military resources they are able to mobilize without jeopardizing their resource and taxation bases. If they stay at the level of this limit, a balance of power emerges because no society has the capability to overcome the others. However, such a balance is not necessarily stable and can degenerate into the tragedy of coercion imagined by Konrad and Skaperdas. If all other factors are held constant, a given society could ratchet up the level of taxation T and thus increase its military power. This will cause the utility level u_i to drop (see (A.3) in the appendix), tighten the budget constraint, and lower the utility function (see Eq. (10)), resulting in Pareto inefficiencies. Feedback effects in the international system place a limit on states' ability to increase taxes in order to increase security and can lead to the tragedy of coercion. The third possibility

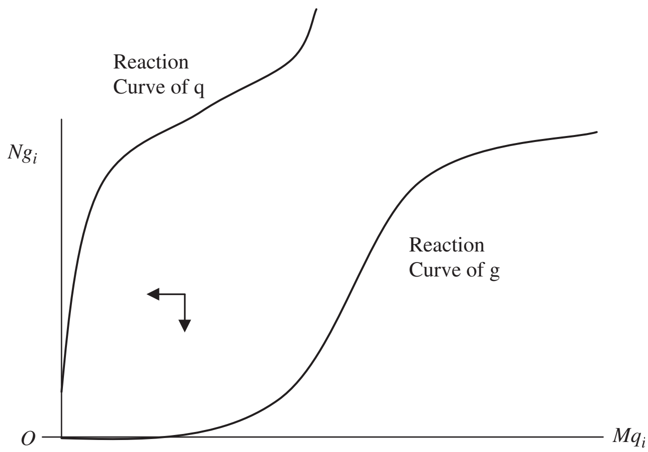

Fig. 4. A Stable Peace.

is a hybrid situation in which some societies increase their military capacities, whereas others only do so marginally. There are two explanations for why some actors remain relatively passive: they are simply less pre-disposed to react, and thus more risk-averse, or they do not have sufficient capability.

The advantage of this theoretical framework is readily apparent. It can both explain how an actor becomes a "Leviathan," in the sense evoked by Konrad and Skaperdas, as well as the "paradox of power" described by Hirshleifer (Hirshleifer, 1991a, 1991b; Konrad & Skaperdas, 1997). On the one hand, we can imagine an equilibrium where society *g* is quickly able to supersede the level of military capability controlled by society *q*. Having reached the limit of its defense capabilities, society *q* can no longer compete with society *g* militarily. What this suggests is that society *g* has in effect become a hegemon, if it can impose itself on all other societies in a similar way, as a "Leviathan." On the other hand, "the initially disadvantaged group is typically rationally motivated to fight harder" according to the "paradox of power," which in our model corresponds to a situation in which a society with a less prominent defense sector has a greater marginal propensity to react and is more risk preferring than the stronger actor (Hirshleifer, 1991a, p. 178). In this case, a more disadvantaged society can overcome a stronger one that is slower to react and is more risk-averse.[17] Vietnam and Iraq come to mind.

Private Goods and Institutions

With the help of our model, we have established that producing private goods in the absence of state structures generates Pareto sub-optimal outcomes and whatever resource is used in the production process will be overdrawn. The tendency of rulers to use more resources than are needed for an optimal system of defense, the "tragedy of coercion," has an immediate counterpart in the tendency of producers to use more resources than what is economically sustainable, i.e., the "tragedy of the commons" (Hardin, 1968). Producers do not take into account the negative externality they impose on others when using a resource. By maximizing average instead of marginal return, they tend to over-exhaust the resource (Dasgupta & Heal, 1979). Not all collective goods are "open access" however. Grouping peasants into villages is a deterrent for bandits and raiders who face a larger threat if a sufficient number of individuals manage to band together. Clearly in this case, each peasant provides a positive externality to other peasants in the community. The medieval city, whose not only size, but also walls and defense organization served to deter brigands and thugs, is a prime example. Eventually, however, negative externalities are bound to appear. If the city becomes densely populated and certain forms of disease start to spread, prospects for collective action will look bleak again. It is thus entirely possible, indeed quite common that a society moves from a situation of increasing to one of diminishing returns to scale, as shown by the S-shaped production curve in Fig. 5 where $F''(Nx)$ is first positive, then negative.

The fundamental problem of collective action is the problem of persuading actors to contribute to the collective good *before* output starts to exceed costs (i.e., before point A in Fig. 5). For production to become profitable in traditional societies, peasants must deter raids; whereas, more advanced societies must establish institutions that will be sufficiently robust to enforce property rights and thereby dissuade other forms of piracy. The logic of sustainability is illustrated in Fig. 5. After point A, average product is higher than marginal product and output greater than costs. A maximum surplus is reached at B, where the slope of the output curve is equal to the slope of the marginal cost curve. Proceeding past point B is not viable. At point C the surplus is completely dissipated resulting in the "tragedy of the commons." The slide to inefficiency is clearly evident, and the collaborative effort transformed from one of soliciting adequate contributions to restricting access. Ensuring that cooperation remains optimal is difficult in a decentralized system. Often the solution is for a dictator, social planner, or hegemon to internalize costs and benefits associated with output in order

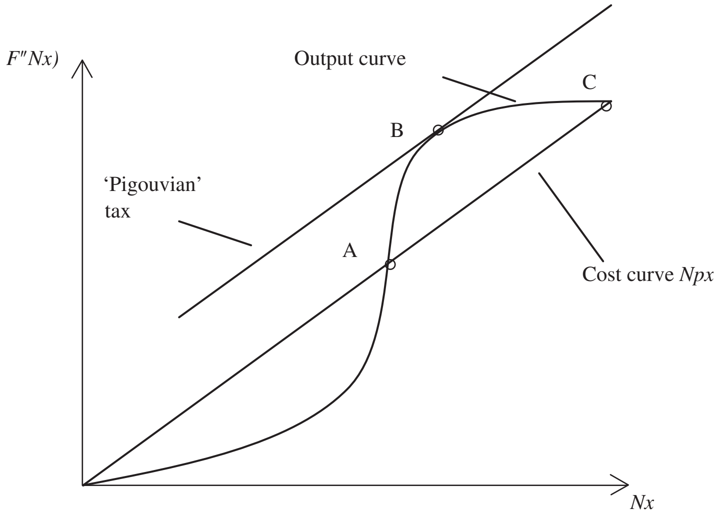

Fig. 5. A Collective Good with Increasing and Decreasing Returns to Scale.

to maximize the surplus at B. Another strategy is to delegate authority over a specific domain in a way that gives each actor an incentive to internalize the constraints of the output curve. Some members of a coalition could, for instance, specialize in military operations and others in diplomatic and humanitarian efforts, all of which would be necessary, to enhance the effectiveness of a security alliance (cf., Boyer, 1989). To secure the viability of agricultural activity, a tract of land could be divided into plots administered by each producer. Finally, Pareto optimality can be attained by introducing a "Pigouvian tax" parallel to the cost curve and tangent to point B (see Fig. 5).

All of these three solutions are "institutional solutions." Implicitly, they assume the provision of a prior collective good in the form of these very institutions. Institutions are thus an inescapable consequence of collaborative efforts that seek to protect society from external encroachment and moreover are central to sustaining a system of production capable of financing defense structures. Even hegemonic provision will at some point involve an institutional setup of some sort to solicit contributions to the good.

Whatever type of institution emerges can be characterized by the degree of coercion involved in sustaining cooperation. Cooperation can occur through relatively open and inexpensive formal structures, which often involve in some form of dispute settlement mechanism. Their function is to preserve and consolidate widespread retaliatory powers and ensure the existence of an S-core. The existence of an S-core, in "prisoner's dilemma," e.g., guarantees the absence of competition for first move.[18] If institutional solutions can be found that substitute for an empty S-core that endows participants with a credible threat to sustain cooperation, there is no reason why conflict and coercion should be pervasive either domestically or internationally. Authoritarian arrangements will not emerge at the domestic level, and at the international level there is no longer the unpleasant choice between an exploitative hegemon and the "tragedy of coercion" that arises in its absence.

Our analysis confronts and contradicts major conclusions of neo-realists who predict that states are programmed to compete with each other on security matters and incapable states will "disappear." We show that this is but one possibility among others (cf., Waltz, 1979). Neo-realists also downgrade the role of economic relationships, but in fact the particular production structure underlying the state matters greatly. States, like firms, occupy particular niches, which promote oligopolistic competition or incentives for trade. Created to provide collective goods, states emerge in environments characterized by external returns to scale (cf., Helpman & Krugman, 1985). Focused on providing collective goods, contiguous societies expend fewer resources fighting each other. As a result, the reduced costs associated with providing the good occurs at the system level. While competition among states may persist, the least competitive ones are not "eliminated," especially when the long-term benefits of conquest are lower than the rewards from other sources of revenue such as trade. In principle therefore, states can coexist with each other and reap the benefits of mutually advantageous trade and a relatively stable international system. Limitations of the neo-realist approach are thus readily apparent. Contrary to neo-realism, which is unable to predict situations in which mutually reinforcing structures for production and protection give way to peace, we are able to derive a theoretically grounded "stable peace," as shown in Fig. 4.

The literature on conflict and exchange, while providing a more sophisticated perspective on the relationship between production and protection, ultimately falls into the same theoretical trap. Its models of society are essentially driven by the relative costs of private versus collective protection and thus primarily by the evolution of military technology (Hirshleifer, 1988, 1989, 1991b; Konrad & Skaperdas, 1997). Both camps have tended to downgrade productive structures. Rather than seeing them as important in their own right, they have almost exclusively viewed them as instrumental in expanding the state's dominion. This pessimism has led to a neglect of stable peace; whereas, the optimistic view taken by liberals has obscured the continuity of different forms of protection. The concentration of power, which is a prerequisite to the enforcement of property rights, is also required to enforce territorial bounds and can under certain circumstances lead to international conflict.

Motives for conquest can be found by looking more closely inside the state. As soon as one acknowledges that the viability of the state hinges upon securing economic activity, one is confronted with the non-trivial problem of how to fund and sustain institutions that protect production. A central authority can finance the common good by eliciting payments through taxes or fees – which can be raised on transactions in numeraire or as the right to labor or to a portion of the good produced – or else by distributing new property rights. In the not-so-distant and remote past, this numeraire often took the form of precious metal. But unless commercial ties are pervasive and transactions monetized, payment for the collective good will most likely be in the goods produced or labor. It is not difficult to see that there are strong incentives on both sides, on the part of the ruled and ruler, to prolong such an arrangement. The authority will try to gain a long-term source of financing by rewarding "loyal" individuals with "permanent use rights," which will be enthusiastically coveted since such rights make it possible to invest and expand production. In the absence of monetized transactions that bring in direct cash payments or institutions that enable rulers to use property as collateral, and thus to borrow, this process creates an incentive for more and more land acquisition (Pirenne, 1970). If conquest is difficult, rulers will eventually lose all source of income, both in the form of short- and long-term payments, to assure defense tasks. According to the famous "Pirenne thesis," this was how the Frankish rulers lost their taxation base and the command of their realm, a decline that was triggered by the interruption of trade in the wake of the Islamic conquests of the seventh and eighth centuries (Pirenne 1925).[19] This logic of decay can also be extended to our own time. Today, societies with weak state structures are ones that are too poor to tax. Often such societies remain "under-developed" because the wider population cannot convert savings into capital without proper institutions that define and protect the right to property against which borrowing can be secured (Soto, 2000). Under these circumstances, states have stronger incentives to provide private goods to a few "well-endowed" individuals rather than collective goods to an impoverished mass (Bueno De Mesquita, Smith, Siverson, & Morrow, 2003). The vicious circle is apparent.

CONCLUSION

This chapter points at the close connection between environmental sustainability, state structure, and coercive power. Our first conclusion is that whether a state is equipped with institutions able to define and enforce property rights is a stronger predictor of the potential for conflict within a society than relative resource abundance. This reconciles two seemingly contradictory positions in the literature on how the size of a country's resource base relates to conflict. Our second finding is that a state that does not strive for Pareto optimality will become weaker and unable to play a significant role at the international level. The power to enforce efficient resource use requires coercive means, which can be used not only to protect resources within states, but also defend and conquer territory from other states. The military power necessary to safeguard and expand a state's resource base requires funding, ideally by taxing productive activity, which brings us back to the importance of optimizing resource use. The collapse of the Soviet Union was due to inefficient property rights protection, inadequate environmental and resource management it implied, and hollowing of the productive base that was supposed to fund its deadly competition with the United States. Although China is at present flush with cash and has for the last two decades increased defense spending, there are already signs it will face major challenges securing the environment. Furthermore, Pareto optimality at the national level can only be completely achieved whenever tragedies of coercion and international arms competition are superseded. As we indicated earlier, this will only occur when incentives to conquer and dominate disappear. Societies will reach their utmost productive potential when states and political groupings realize that more wealth can be generated by fostering and maintaining trade and other international transactions than by violent appropriation.

NOTES

1. Competing lords would, however, also have to fight each other and seek to dominate as many peasants as possible. This can lead to what Konrad and Skaperdas call a ''Tragedy of Coercion,'' a sub-optimal Nash equilibrium outcome, where resources are wasted in the mutual fights for the control of peasants.

2. Hirshleifer's perspective is even more driven by military technology. See for instance his decisiveness function (Hirshleifer, 1988, 1989, 1991b). Even though, Hirshleifer, Konrad, and Skaperdas and others bring very new and interesting insights into the international conflict and cooperation literature, their fundamental assumptions are very close to the ones of neo-realism.

3. cf. Johnson and Earle (2000) and to some extent Diamond (1997).

4. For a significant departure and criticism of the realist position, the image of the international world it projects, and for an emphasis upon trade, see the interesting book by Rosecrance (1986).

5. Friedman and Savage (1948) use a perspective on their utility function in their article that differs markedly from ours.

6. The S-curve analysis and its application to conflict was initiated by Dacey (1996a, 1996b, 1996c, 1998) and Dacey and Gallant (1997). The formulation used here below for the critical risk ratio is based on losses, whereas the formulation used in Dacey is on gains. These formulations are logically equivalent. The S-curve hypothesis is just another illustration that attitudes toward risk are implicitly included in any representation of a utility function because they are implied in its particular curvature. Even a linear (flat) utility function results in risk neutrality. What we are saying here is that the shape of the utility curve implies different behavioral consequences. The choice of a representation in terms of risk neutrality (a flat curve) or risk aversion or a mix of both such as the one we are presenting here is thus not a matter of theoretical debate but ultimately an empirical question. It seems that empirical results are still contested despite the cogency of the psychological literature that we are citing. But even if the ultimate empirical test on this issue has not yet been presented, the validity of the mixed approach at the theoretical level in our view is indisputably a promising formulation that leads to richer behavioral consequences than classical (risk neutral or risk averse) notions. Some of the results presented here could be obtained with a more classical utility function, as in Dasgupta and Heal (1979), 41 but are not as rich as the ones we obtain.

7. From different premises, evolutionary game theorists arrive at the same conclusion, cf., Binmore (1998), Chapter 2, particularly pp. 204–211.

8. This is also true for the utility function used by Dasgupta and Heal (1979).

9. *Ibid.*, p. 43 do not establish any such link.

10. *Ibid.*, p. 44–52.

11. Provided only positive values for the terms under the square root signs are considered.

12. See Tilly (1992) in particular emphasizes this point.

13. This point is made by Pirenne (1925).

14. After the British economist Pigou (1932).

15. If $t_i < 0$, the subsidy is in fact a tax, and if $\tau < 0$, the lump-sum tax becomes a subsidy. Dasgupta and Heal (1979).

16. Such minimal ratios can be interpreted as Lanchester (1916) combat ratios (see appendix).

17. All these statements can be supported rigorously within the model. Since we are not focusing here on armament and arms race questions, we refer the reader to the appendix.

18. Formally, an S-core is non-empty if each agent can reach a utility level at least as great as the one reached by moving second in an asymmetric Nash equilibrium such as the Chicken or Stackelberg equilibrium, cf. Moulin (1986).

19. For a critical view see McCormick (2002). Although according to Greif (1993) even in the eleventh century, commercial contacts between the Jewish merchants in the Moslem and the Christian regions were non-existent, suggesting that Pirenne was right.

ACKNOWLEDGMENTS

We would like to thank Steven Bernstein, Lars-Erik Cederman, Xinyuan Dai, James Morrow, Martin Osborne, Peter Rosendorff, and Duncan Snidal for their helpful comments.

REFERENCES

Battalio, R. C., Kagel, J. H., & Jiranyakul, K. (1990). Testing between alternative models of choice under uncertainty: Some initial results. *Journal of Risk and Uncertainty*, *3*, 25–50.

Battalio, R. C., Kagel, J. H., & MacDonald, D. N. (1985). Animals' choices over uncertain outcomes: Some initial experimental results. *American Economic Review*, *75*, 597–613.

Binmore, K. (1998). *Game theory and the social contract: Just playing* (Vol. II). Cambridge: MIT Press.

Boyer, M. A. (1989). Trading public goods in the western alliance system. *The Journal of Conflict Resolution*, *33*(4), 700–727.

Bromley, D. W., & Feeny, D. (1992). *Making the commons work: Theory, practice, and policy*. San Francisco: ICS Press.

Bueno De Mesquita, B., Smith, A., Siverson, R. M., & Morrow, J. D. (2003). *The logic of political survival*. Cambridge: The MIT Press.

Camerer, C. F. (1989). An experimental test of several generalized utility theories. *Journal of Risk and Uncertainty*, *2*, 61–104.

Collier, P., & Hoeffler, A. (1998). On economic causes of civil war. *Oxford Economic Papers*, *50*, 563–573.

Conybeare, J. (1980). International organization and property rights. *International Organization*, *34*(3), 307–334.

Dacey, R. (1996a). Dissatisfaction and the illicit response. Paper presented at the Fifth world peace science society congress. Amsterdam.

Dacey, R. (1996b). Dissatisfaction and the illicit response. Paper presented at the Peace science society. Houston.

Dacey, R. (1996c). The maintenance of authority: Risk attitude and the deterrence of civil disobedience. Paper presented at the American economic association. San Francisco.

Dacey, R. (1998). Risk attitude, punishment, and the Intifada. *Conflict Management and Peace Science*, *16*, 77–88.

Dacey, R., & Gallant, K. (1997). Crime control and harassment of the innocent. *Journal of Criminal Justice*, *25*, 325–334.

Dasgupta, P., & Heal, G. (1979). *Economic theory and exhaustible resources*. Cambridge: Cambridge University Press.

Demsetz, H. (1967). Toward a theory of property rights. *American Economic Review*, *57*(2), 347–359.

Diamond, J. (1997). *Guns, germs, and steel*. New York: Random House.

Fishburn, P. C., & Kochenberger, G. (1979). Two-piece von Neumann-Morgenstern utility functions. *Decision Sciences*, *10*, 503–518.

Friedman, M., & Savage, L. J. (1948). The utility analysis of choices involving risk. *Journal of Political Economy*, *56*, 279–304.
Gilpin, R. (1981). *War and change in world politics*. Cambridge: Cambridge University Press.
Greif, A. (1993). Contract enforceability and economic institutions in early trade: The Maghribi traders' coalition. *American Economic Review*, *83*(3), 525–548.
Grossman, H. (1991). A general equilibrium model of insurrections. *American Economic Review*, *81*(4), 919–921.
Hardin, G. (1968). The tragedy of the commons. *Science*, *162*, 1243–1248.
Helpman, E., & Krugman, P. R. (1985). *Market structure and foreign trade: Increasing returns, imperfect competition, and the international economy*. Cambridge: MIT Press.
Hirshleifer, J. (1988). The analytics of continuing conflict. *Synthese*, *76*, 201–233.
Hirshleifer, J. (1989). Conflict and rent-seeking success functions: Ratio vs. difference models of relative success. *Public Choice*, *63*, 101–112.
Hirshleifer, J. (1991a). The paradox of power. *Economics and Politics*, *3*(3), 177–200.
Hirshleifer, J. (1991b). The technology of conflict as an economic activity. *American Economic Review*, *81*(2), 130–134.
Hirshleifer, J. (1995). Anarchy and its breakdown. *Journal of Political Economy*, *103*(11), 26–50.
Homer-Dixon, T. (1994). Environmental scarcities and violent conflict: Evidence from cases. *International Security*, *19*(1), 5–40.
Johnson, A. W., & Earle, T. (2000). *The evolution of human societies*. Stanford: Stanford University Press.
Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decisions under risk. *Econometrica*, *47*, 263–291.
Konrad, K. A., & Skaperdas, S. (1997). *The market for protection and the origin of the state*. Irvine: Free University of Berlin.
Lanchester, F. W. (1916). *Aircraft in warfare, the dawn of the fourth arm*. London: Constable.
Luterbacher, Urs. (1994). International cooperation: The problem of the commons and the special case of the Antarctic region. *Synthese*, *100*, 413–440.
McCormick, M. (2002). *Origins of the European economy: Communications and commerce, Ad 300–900*. New York: Cambridge University Press.
Moulin, H. (1986). *Game theory for the social sciences. 2nd and rev. edition, Studies in game theory and mathematical economics*. New York: New York University Press.
Netting, R. McC. (1981). *Balancing on an Alp*. Cambridge: Cambridge University Press.
North, D. C. (1981). *Structures and change in economic history*. New York: W.W. Norton.
Olson, M. (1965). *The logic of collective action*. Cambridge: Harvard University Press.
Olson, M. (1982). *The rise and decline of nations*. New Haven: Yale University Press.
Ostrom, E. (1990). *Governing the commons: The evolution of institutions for collective action*. Cambridge: Cambridge University Press.
Pigou, A. C. (1932). *The economics of welfare*. London: Macmillan.
Pirenne, H. (1925). *Medieval cities: Their origins and the revival of trade*. Princeton: Princeton University Press.
Pirenne, H. (1970). *Mohomet et Charlemagne*. Paris: Presses Universitaires de France.
Rosecrance, R. (1986). *The rise of the trading state*. New York: Basic Books.
Sandler, T. (1992). *Collective action: Theory and applications*. Ann Arbor: University of Michigan Press.
Soto, H. de. (2000). *The mystery of capital*. London: Random House.

Stephens, D. W. (1990). Risk and incomplete information in behavioral ecology. In: C. Elizabeth (Ed.), *Risk and uncertainty in tribal and peasant economies*. Boulder: Westview.
Stevenson, G. G. (1991). *Common property economics*. Cambridge: Cambridge University Press.
Tilly, C. (1992). *Coercion, capital, and European states, Ad 990–1992*. Cambridge: Basil Blackwell.
Usher, D. (1989). The dynastic cycle and the stationary state. *American Economic Review*, *79*, 1031–1044.
Waltz, K. N. (1979). *Theory of international politics*. Reading: Addison-Wesley.
Wiegandt, E. (1977). Inheritance and demography in the Swiss Alps. *Ethnohistory*, *24*(2).

APPENDIX. TECHNICAL APPENDIX

The Society Market or Anarchic Equilibrium

Optimization yields: $\mathrm{Max}\{\exp\{\alpha - 1/x_i - 1/[g_i + (N-1)\hat{g}]\} + \lambda_i(1 - p^s g_i - x_i)\}$ Thus first-order conditions will be:

$$\frac{1}{x_i^2}\exp\left\{\alpha - \frac{1}{x_i} + \frac{-1}{[g_i + (N-1)\hat{g}]}\right\} = \lambda_i \tag{A.1}$$

$$\frac{1}{\left[g_i + (N-1)\hat{g}\right]^2}\exp\left\{\alpha - \frac{1}{x_i} + \frac{-1}{[g_i + (N-1)\hat{g}]}\right\} = \lambda_i p^s \tag{A.2}$$

For agent i, the problem is to maximize: $\exp\{\alpha - 1/x_i - 1/[g_i + (N-1)\hat{g}]\}$ by choosing x_i and g_i subject to the budget constraint Eq. (4). The necessary (and eventually sufficient since the utility function will after being initially convex become concave) conditions for an optimum will be:

$$\mathrm{Max}\left\{\exp\left\{\alpha - \frac{1}{x_i} - \frac{1}{[g_i + (N-1)\hat{g}]}\right\} + \lambda_i(1 - p^s g_i - x_i)\right\}$$

At the *anarchic equilibrium*, one can assume that $g_i = \hat{g}$ and thus $x_i = \hat{x}$.
From the first-order conditions, we therefore have, $N\hat{g} = \hat{x}/\sqrt{p^s}$ which using the budget constraint Eq. (4), gives for $\hat{x}$ and $\hat{g}$ (see Eq. (5))

Comparative Statics: Anarchic and Pareto Optimal Equilibrium

To answer the question of whether the anarchic equilibrium is Pareto efficient one has to treat g as if it were another kind of private good and considered by agent i as if she was alone and thus maximizes: $\exp\{\alpha - 1/x - 1/N_g\}$ subject to the budget constraint $p^s g + x \leq 1.4$.

The Pareto optimal solution $(\tilde{x}, \tilde{g})$ can be found readily as in Eq. (6).

Quite clearly, the anarchic equilibrium is not Pareto optimal.

To show that the anarchic equilibrium is sub-optimal, one has to compare the exponents in the utility function $\exp\{\alpha - 1/x - 1/Ng\}$. The higher the expression within the parenthesis (inside the curly bracket), the lower is the value of the utility function $\exp\{\alpha - 1/x - 1/Ng\}$. Therefore, the value of the function with the Pareto optimal solution $(\tilde{x}, \tilde{g})$ should be superior to the anarchic solution and thus the exponent value within the parenthesis inside the curly bracket of the exponent is lower than the one for the anarchic solution, which is indeed the case. To see that the value of the function with the Pareto optimal solution $(\tilde{x}, \tilde{g})$ is superior to the anarchic solution compare:

$$\frac{\left(\sqrt{p^s} + \sqrt{N}\right)^2}{N} \text{ the Pareto optimal value}$$
$$< \frac{\left(\sqrt{p^s} + N\right)\left(\sqrt{p^s} + 1\right)}{N} \text{(the anarchic solution value), for all} \quad N > 1$$

Pareto Optimal Equilibrium: Taxation

Agent i in the absence of any market for externalities maximizes:

$$u_i\left(x_i, \sum_{j=1}^{N} g_i\right) = \exp(\alpha - \frac{1}{x_i} + \frac{-1}{\Sigma_{j\neq 1}^{N} g_j + g_i} \text{subject to } (p^s - t)g_i + x_i \leq 1 - \tau_i \tag{A.3}$$

and where of course agent i chooses only x_i and g_i. By analogy with the previous results, we get at equilibrium, assuming that $p^s = p^s - t$:

$$Ng = \frac{x}{\sqrt{p^s}} \tag{A.4}$$

To get to the Pareto optimal result (6) with, $\tilde{g} = \tilde{x}/\left(\sqrt{N}\sqrt{p^s}\right)$, the net price p^s that an agent must pay for the externality should be $p^s = p^s/N = Np^s/N^2$. Indeed, introducing this expression into (A.3) leads to the Pareto optimal value (6) restated above. Thus the authority must set the per unit subsidy of the collective good at $t = (N-1)p^s/N$. The authority must also set a lump-sum tax on each agent again with the purpose to reach Pareto optimality as defined by the values of $\tilde{x}$ and $\tilde{g}$ in (6). This lump sum tax τ, is thus:

$$\tau = \frac{\sqrt{p^s}(N-1)}{N\left(\sqrt{p^s}+\sqrt{N}\right)} \tag{A.5}$$

On this basis, one is now able to compute the total authority expenditures and revenues. Total expenditures or subsidies for the collective good are identical (see expression Eq. (8))

Coalition Size

The optimal size is given by:

$$\frac{\partial N\tau}{\partial N} = \partial \frac{\left((N-1)\sqrt{p^s}\right)/\left(\sqrt{p^s}+\sqrt{N}\right)}{\partial N} = 0$$

The solution (for a maximum) eventually leads to:

$$N = 2\sqrt{p^s}\left(\sqrt{p^s}\sqrt{p^s 1}\right) - 1 \text{ or } 2\sqrt{p^s}\left(\sqrt{p^s}\sqrt{p^s 1}\right) - 1$$

with the second solution leading to higher values.

Optimal Reaction Functions

Recall that $\overline{\overline{Ng_i}}$ is determined by $(\overline{Ng_i})/(a\sqrt{(k_g/k_q)} + b(r_g/r_q)) \geq Mq_i$, where Mq_j represents the total size of the other society's q's defense sector, in utility function (10). Mq_i can thus directly be included in the utility function of i, which can be written as:

$$u_i\left(x_i, \sum_{j=1}^{N} g_i,\right) = \exp\left(\alpha - \frac{1}{x_i} - \frac{1}{\Sigma_{j\neq 1}^{N} g_j + g_i} - \frac{1}{Mq_i}\right) \tag{A.6}$$

This utility function has to be maximized subject to:

$$(p^s - t)g_i + x_i \leq 1 - \tau_i \text{ and } Ng_i \geq \left(a\sqrt{\frac{k_g}{k_q}} + b\frac{r_g}{r_q}\right)Mq_i$$

$$\text{or } \frac{Ng_i}{\left(a\sqrt{(k_g/k_q)} + b(r_g/r_q)\right)} \geq Mq_i \tag{A.7}$$

which leads to the following expression to be maximized:

$$\exp\left(\alpha - \frac{1}{x_i} - \frac{1}{Ng_i} - \frac{1}{Mq_i}\right) + \left[1 - \tau - x_i - g_i(p^s - t)\right] l + \left[\frac{Ng_i}{\left(a\sqrt{(k_g/k_q)} + b(r_g/r_q)\right)} - Mq_i\right] m \tag{A.8}$$

where l and m are the two Lagrange multipliers associated with the constraints. These multipliers can also be interpreted as the respective shadow prices associated with (1) the internal budget constraint (l) and (2) the defense effort with respect to society q (m).
One gets the equations:

$$\frac{1}{x_i^2}\exp\left(\alpha - \frac{1}{x_i} - \frac{1}{Ng_i} - \frac{1}{Mq_i}\right) - l = 0 \tag{A.9}$$

$$\frac{1}{Ng_i^2}\exp\left(\alpha - \frac{1}{x_i} - \frac{1}{Ng_i} - \frac{1}{Mq_i}\right) - [p^s - t]l + \left[\frac{mN}{\left(a\sqrt{(k_g/k_q)} + b(r_g/r_q)\right)}\right] = 0 \tag{A.10}$$

Replacing l by its value from (A.8), one gets:

$$\left[\frac{1}{Ng_i^2} - \frac{p^s - t}{x_i^2}\right]\exp\left(\alpha - \frac{1}{x_i} - \frac{1}{Ng_i} - \frac{1}{Mq_i}\right) + \left[\frac{mN}{\left(a\sqrt{(k_g/k_q)} + b(r_g/r_q)\right)}\right] = 0 \tag{A.11}$$

Rearranging terms and remembering that m is by definition an arbitrary constant:

$$\exp\left(\alpha - \frac{1}{x_i} - \frac{1}{Ng_i} - \frac{1}{Mq_i}\right) = -\left[\frac{mN}{\left(a\sqrt{(k_g/k_q)} + b(r_g/r_q)\right)}\right]\frac{x_i^2 Ng_i^2}{-Ng_i^2 p^s + Ng_i^2 t + x_i^2} \tag{A.12}$$

If x_i is given its Pareto optimal value in terms of g_i, $\tilde{g} = \tilde{x}/\sqrt{N}\sqrt{p^s}$ and thus $x_i = \sqrt{N}\sqrt{p^s}g_i$, and if t is taken as a tax$-t = T$ rather then a subsidy, (a possibility evoked when defining t and τ) which makes more sense in terms of explaining expenditures on the collective defense good, then Eq. (A.12) reduces to:

$$\frac{\left(a\sqrt{(k_g/k_q)}+b(r_g/r_q)\right)T}{mp^s}\exp\left(\alpha-\frac{1}{x_i}-\frac{1}{Ng_i}-\frac{1}{Mq_i}\right)=N^2g_i^2 \qquad \text{(A.13)}$$

or if only positive roots are taken:

$$\left[\frac{\left(a\sqrt{(k_g/k_q)}+b(r_g/r_q)\right)T}{mp^s}\right]^{1/2}\exp\frac{1}{2}\left(\alpha-\frac{1}{x_i}-\frac{1}{Ng_i}-\frac{1}{Mq_i}\right)=Ng_i \quad \text{(A.14)}$$

Expression (A.14) determines thus, among other things the optimal reaction of society g in terms of its defense sector Ng_i with respect to Mq_i, the defense sector of society q, which for simplicity can be expressed as expression Eq. (11).